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LAMINAR BOUNDARY LAYER MASS TRANSPORT FROM A DISC ROTATING IN A VISCOELASTIC FLUID: SIGNIFICANCE IN DIFFUSIVITY MEASUREMENTS

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NOMENCLATURE*

$a'(n)$, function of n in equation (18);
D, molecular diffusion coefficient;
$e^{1(ik)}$, rate of deformation tensor;
F, dimensionless radial velocity;
f_1, f_2, functions of Wi defined in equations (12) and (13);
G, dimensionless rotational velocity;
g_{ik}, matrix tensor of a fixed co-ordinate system;
H, dimensionless axial velocity;
$j(r)$, local mass flux;
$J(\Lambda)$, function defined in equation (9);
k, mass transfer coefficient;
K, consistency index;
n, power-law index;
$N(\tau)$, distribution function of relaxation times;
p, arbitrary isotropic pressure;
p_{ik}, stress tensor;
p'_{ik}, deviatoric stress tensor;
r, local radius;
R, radius of disc;
Re, Reynolds number ($= R^2\Omega/\nu$);
Sh_0, Sherwood number for Newtonian fluid ($= kR/D$);
Sh^*, Sherwood number for viscoelastic fluid;
Wi, Weissenberg number ($= \lambda\Omega/\eta_0$);
z, axial distance.

Greek letters

$\beta(n)$, function of n in equation (17);
$\delta/\delta t$, convected time derivative;
η_0, zero shear viscosity,
θ, dimensionless concentration;
λ, relaxation time function;
Λ, Schmidt number ($= \nu/D$);
ν, kinematic viscosity;
ξ, dimensionless axial distance;
ρ, density;
τ, relaxation time;
τ_ω, local shear stress on the surface of the disc;
τ_{avg}, average shear stress on the surface of the disc;
Ω, angular velocity.

THE PROBLEMS concerned with the diffusive transport rates in macromolecular solutions have received some attention in recent years due to their obvious pragmatic significance. It is well known that for such macromolecular solutions the changes in molecular alignment occurring in a shear field alter the momentum transfer characteristics quite significantly. The perplexing problem of the influence of a shear field on the diffusion coefficient of a low molecular weight solute in such solutions can be answered satisfactorily only if such measurements were done under flow conditions with well defined shear rate levels in the region which is controlling the diffusive transport rates.

One of the most widely used techniques for studying the convective mass and heat transport under well defined hydrodynamic conditions is the rotating disc apparatus (see Levich [1], for instance). This technique has been used for obtaining the diffusivities of solutes in dilute polymeric solutions by Hansford and Litt [2], Luikov *et al.* [3] and recently by Greif *et al.* [4]. The diffusivities were deduced on the basis of the theoretical solution of the convective diffusion equation. The latter was specifically obtained for transport from a disc rotating under laminar boundary layer flow conditions in *inelastic* power law fluids. The neglect of elasticity may not be strictly justified since all the polymer solutions used by these authors are known to possess significant levels of elasticity as evidenced by the presence of the secondary flow patterns under the experimental conditions as well as by the presence of the finite normal differences which these solutions are known to exhibit under viscometric flow conditions. Further recent work of Kale *et al.* [5] shows clearly that the momentum transfer characteristics of the rotating disc laminar boundary layer flows are influenced significantly by the presence of elasticity. Since the rotating disc technique is likely to be used more extensively in the future it is important to examine the influence of elasticity, if any, on the rate of mass transport from the disc. This note assesses the problem theoretically and also draws attention to some important conclusions which can be drawn by performing rotating disc experiments, which are otherwise either difficult or impossible to draw.

Elliott [6] has obtained the velocity distribution around a disc rotating in a viscoelastic liquid designated as Walters B′ liquid. The constitutive equations characterizing this liquid are given by Walters [7]. In the case of liquids with short memories or short relaxation times the constitutive equation is written in the form:

$$p_{ik} = -pg_{ik} + p'_{ik} \tag{1}$$

* Any set of consistent units may be used.

with

$$p'_{ik} = 2\eta_0\, e^{(1)ik} - 2\lambda \frac{\delta\, e^{(1)ik}}{\delta t} \qquad (2)$$

where

$$\eta_0 = \int_0^\infty N(\tau)\, d\tau \qquad (3)$$

and

$$\lambda = \int_0^\infty \tau N(\tau)\, d\tau. \qquad (4)$$

Here p_{ik} is the stress tensor, p is an isotropic pressure, g_{ik} is the metric tensor of a fixed co-ordinate system x^i, $e^{1(ik)}$ is the rate of deformation tensor, η_0 is the zero shear viscosity and $N(\tau)$ is the distribution function of relaxation times τ. $\delta/\delta t$ is the convected derivative. This fluid shows under viscometric flow conditions a constant viscosity and finite primary normal stress differences which vary as the square of the shear rate. The limitations of such second order approximation of fluid behaviour have been discussed by Mashelkar *et al.* [8] and Kelkar *et al.* [9]. However the boundary layer approximations appear to be justified for such a fluid and the influence of elasticity on the rate of mass transport can be studied at least semi-quantitatively by using such fluid behaviour description.

The relevant convective diffusion equation for the case of a disc rotating under laminar boundary layer flow conditions can be shown to reduce to (10):

$$\frac{\partial^2 \theta}{\partial \xi^2} = \Lambda H(\xi) \frac{\partial \theta}{\partial \xi}. \qquad (5)$$

Elliott has obtained the expression for $H(\xi)$ in the following form:

$$H(\xi) = H_0(\xi) + Wi\, H_1(\xi). \qquad (6)$$

Here Wi is the Weissenberg number defined as $Wi = \lambda\Omega/\eta_0$. Equation (6) may be expanded in terms of the MacLaurin series. Using the expressions for F_0, F_1, G_0, G_1, H_0 and H_1 reported by Elliott and also making use of the equations of continuity and motion we obtain the expansion up to the first three terms,

$$H(\xi) = -(1{\cdot}02 + 0{\cdot}602Wi)\frac{\xi^2}{2!} + (2 + 1{\cdot}5176Wi)\frac{\xi^3}{3!} - (2{\cdot}464 + 5{\cdot}288Wi)\frac{\xi^4}{4!}. \qquad (7)$$

Solution of equation (5) with the boundary conditions $\xi = 0$, $\theta = 1$, $\xi \to \infty$, $\theta = 0$ will give the concentration distribution, diffusion flux and the Sherwood number (Sh^*) as:

$$Sh^* = Re^{\frac{1}{2}}/J(\Lambda) \qquad (8)$$

where

$$J(\Lambda) = \int_0^\infty \exp\{\Lambda \int_0^\xi H(\xi)\, d\xi\}\, d\xi. \qquad (9)$$

Using equation (7) we obtain

$$\Lambda \int_0^\xi H(\xi)\, d\xi = \Lambda\left\{-(1{\cdot}02 + 0{\cdot}602Wi)\frac{\xi^3}{3!} + (2 + 1{\cdot}5176Wi)\frac{\xi^4}{4!} - (2{\cdot}464 + 5{\cdot}288Wi)\frac{\xi^5}{5!}\right\}. \qquad (10)$$

Setting $\Lambda(1{\cdot}02 + 0{\cdot}602Wi)(\xi^3/3!) = x^3$, inserting in equation (10) and rearranging after substitution in equation (9) we get

$$J(\Lambda) = \left(\frac{3!}{1{\cdot}02 + 0{\cdot}602Wi}\right)^{\frac{1}{3}} \Lambda^{-\frac{1}{3}} \int_0^\infty \exp(-x^3) \times \exp\left[\left(f_1 \frac{(3!)^{\frac{4}{3}}}{4!}\Lambda^{-\frac{1}{3}}x^4\right) - \left(f_2 \frac{(3!)^{\frac{5}{3}}}{5!}\Lambda^{-\frac{2}{3}}x^5\right)\right] \qquad (11)$$

where

$$f_1 = (2 + 1{\cdot}5176Wi)/(1{\cdot}02 + 0{\cdot}602Wi)^{\frac{4}{3}} \qquad (12)$$

$$f_2 = (2{\cdot}464 + 5{\cdot}288Wi)/(1{\cdot}02 + 0{\cdot}602Wi)^{\frac{5}{3}}. \qquad (13)$$

Following Newman [11] and Ke'tien and Stewart [12], the second exponential is now expanded, terms with identical powers of Λ are rearranged and each term is evaluated as a gamma function whereupon we obtain

$$Sh^* = Re^{\frac{1}{2}}(1{\cdot}02 + 0{\cdot}602Wi)\Lambda^{\frac{1}{3}}/[1{\cdot}6439 + 0{\cdot}2401 f_1 \Lambda^{-\frac{1}{3}} + 0{\cdot}1248(f_1^2 - 0{\cdot}8235 f_2)\Lambda^{-\frac{2}{3}}]. \qquad (14)$$

Equation (14) predicts Sh^* for a viscoelastic fluid as a function of Re, Wi and Λ. For a Newtonian fluid ($Wi = 0$) equation reduces to the corresponding expression given by Ke'Tien and Stewart [12] when expansion only up to the first three terms is considered.

The influence of elasticity is now examined by calculating the ratio of Sh^* (for viscoelastic fluid) and Sh_0 (for Newtonian fluid). Calculations at identical Re indicate that there is a marginal enhancement in mass transfer with increasing Wi (corresponding to increasing elasticity). Thus at $Wi = 0{\cdot}05$, 0·25 and 0·5; respectively, Sh^* is found to be increased by 1·3, 4·7 and 9 per cent; respectively. Since for large Λ an excellent approximation is given by $Sh^* \propto \Lambda^{\frac{1}{3}}$, this would indicate that the level of errors in the measured values of diffusivities when the presence of elasticity is neglected could be given approximately by 2, 7 and 13·5 per cent; respectively. It is of course hazardous to draw conclusions for large Wi, since the limitation of the second order approximation mentioned earlier implies validity of these conclusions only for small values of Wi.

An important point of possible practical significance emerges from this analysis. The addition of small amount of certain polymers is known to reduce the friction factors in internal flows and the moment coefficients in the external rotational flows under turbulent flow conditions [13]. This addition, however, results in either a proportional decrease [14] or a more than proportional decrease [15, 16] in heat or mass transfer coefficients. The work of Kale *et al.* [5] has shown that for the case of a disc rotating in dilute polymer solutions there is a significant reduction in frictional resistance even under *laminar* boundary layer flow conditions. The accompanied increase in heat or mass transfer rates under the same conditions thus appears to suggest a significant advantage.

When flow techniques are employed, the influence of shear rate on the diffusion coefficient can be comfortably studied for the case of solid dissolution since the shear rate in the region of thin concentration boundary layer can be assumed to be finite, well defined and approximately constant [17, 18]. For the case of gas absorption, however, this does not appear to be the case (Mashelkar and Soylu [19]). In this case the stagnant media techniques (e.g. [20]) are unsuitable (no flow). Laminar jet technique (e.g. [21]) is also unsuitable (assumed flat velocity profile). Other model apparatus such as a wetted wall [22] or a wetted sphere column [19] also appear to be not very convenient. In such cases the gas absorption occurs from the stress free gas–liquid interface and the "depth of penetration" of the gas is usually very small. The presence of the sheared region inside the surface does not substantially influence the diffusive transport. The rotating disc technique as used by Greif *et al.* appears to be particularly suitable for such a study since the diffusional boundary layer is confined to a very thin zone near the disc surface which is the most sheared region and has finite well defined shear rates. If the liquid is assumed to be inelastic as a simplification then the shear stress at the surface of the disc can be shown to be given by

$$\tau_\omega = K\Omega^{3n/(n+1)}\left(\frac{K}{\rho}\right)^{-1/(n+1)} r^{2n/(n+1)} \times [F'(0)^2 + G'(0)^2]^{(n-1)/2} F'(0). \quad (15)$$

Using the expressions for F and G calculated by Mitschka and Ulbrecht [23], we can calculate the local shear stress on the surface of the disc as

$$\tau_\omega = \beta(n) K\Omega^{3n/(n+1)}\left(\frac{K}{\rho}\right)^{-1/(n+1)} r^{2n/(n+1)} \quad (16)$$

where

$$\beta(n) = 0{\cdot}1539\left(\frac{5n+3}{n+1}\right)(6{\cdot}13)^{(n-1)/[2(n+1)]}. \quad (17)$$

It is interesting to note that for a power-law fluid the disc is no more uniformly accessible and the local mass transfer flux is given by [2]

$$j(r) = \left(\frac{a'(n)}{3}\right)^{\frac{1}{3}}\left[\frac{C_0 D^{\frac{2}{3}}}{\frac{1}{3}\Gamma(\frac{1}{3})}\right]\left(\frac{K}{\rho}\right)^{-\frac{1}{3}[1/(1+n)]} r^{\frac{1}{3}[(1-n)/(1+n)]}\Omega^{1/(1+n)}. \quad (18)$$

Since τ_ω varies as $r^{2n/(n+1)}$ and $j(r) \propto r^{\frac{1}{3}[(1-n)/(1+n)]}$ it appears that the electrochemical technique used by Greif *et al.* can be suitably used for studying the dependence of the diffusivities on the shear rate or the shear stress. It will be necessary to perform experiments only at a fixed rotational speed but at different activated circular ring portions on the disc. By keeping the ring portions reasonably small, a constant shear rate can be ensured.

With the foregoing analysis it becomes clear that the work of Greif *et al.* is the only work in the literature from which the influence of shear stress on the diffusivity of gaseous solutes in polymer solutions can be reliably deduced. From equation (16) the area averaged shear stress on the disc surface is obtained as

$$\tau_{avg} = \left(\frac{n+1}{2n+1}\right)\beta(n) K\Omega^{3n/(n+1)}\left(\frac{K}{\rho}\right)^{-1/(n+1)} R^{2n/(n+1)}. \quad (19)$$

Thus τ_{avg} is a strong function of the angular velocity ($\tau_{avg} \propto \Omega^{3n/(n+1)}$). The analysis of the data of Greif *et al.* indicated that over approximately 50 fold change in τ_{avg} the diffusion coefficient remained constant. It is interesting to note here the striking contrast in the results obtained by Greif *et al.* and Wasan *et al.* [22] for polyox-WSR-301–O_2 system. Wasan *et al.* used a wetted wall column and obtained a very strong dependence of diffusivity on the shear rate at the wall. Since the work of Greif *et al.* provides the most sensitive test of the shear rate dependence of diffusivity, the conclusions of Wasan *et al.* hence appear to be somewhat doubtful. Recent work of Mashelkar and Soylu [19] with wetted sphere column does not also lend support to the type of observations reported by Wasan *et al.* More experimental work (perhaps in the directions indicated in this work) is needed before such discrepancies may be completely resolved.

CONCLUSION

A theoretical analysis of the mass transport from a disc rotating under laminar boundary layer flow conditions into a Walters B′ liquid indicates that elasticity enhances the transport rate to some extent. This finite influence may have some effect on the deduced values of diffusivities. Rotating disc technique as used by Greif *et al.* appears to be more suitable for studying the shear rate dependence of diffusivities of gaseous solutes than many of the existing model apparatus. The analysis of the work of Greif *et al.* indicates a shear rate independent diffusion coefficient.

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THERMAL PROPERTIES OF KRYPTON AND XENON LAMINAR ARC PLASMAS

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INTRODUCTION

IN SUPPORT of industrial and research applications of the cascade arc plasma, considerable effort has been devoted to the theoretical determination of arc thermal properties. Although emphasis has been placed on the argon arc, some consideration has been given to hydrogen, helium and nitrogen arcs as well. For this purpose numerous equilibrium models have been developed, the most rigorous of which is that due to Bower and Incropera [1]. Although more recent work has revealed the existence of thermochemical non-equilibrium effects [2, 3], the equilibrium model has been found suitable for many engineering purposes. In particular, results obtained from this model suggest important trends and provide excellent reference conditions for comparison with data and more refined non-equilibrium calculations.

In this study the equilibrium model of Bower and Incropera [1] has been extended to krypton and xenon arc plasmas operating in a laminar mode. Interest in these gases has been stimulated by their intense radiation characteristics. Since krypton and xenon are more readily ionized than most other arc gases, they provide potentially excellent sources of both visible and ultraviolet radiation for photochemical processing.

FLOW MODEL AND RESULTS

The flow model used in this study is identical to that developed by Bower and Incropera [1] and is based upon the assumption of local thermochemical equilibrium. Although calculations have been performed for both the entrance and asymptotic arc regions, results presented in this article are restricted to the asymptotic region. In this region, flow properties depend only upon radial location in the arc (for prescribed arc operating parameters), and the dissipation of electrical energy is identically balanced by convective and radiative losses. The transport coefficients required for the calculations were obtained from Devoto [4], and the thermodynamic properties were computed from statistical mechanics [5]. The radiation source term required for the model was determined from microscopic considerations which accounted for both continuum and line contributions. Details are provided by Greene [5].

Parametric calculations were performed in which the arc operating parameters (pressure, current and radius) were varied over wide ranges. Dependent variables of particular interest include the temperature profiles, the total wall heat flux, and the relative contribution of radiation to this heat flux.

Temperature profiles computed for a range of currents are shown in Figs. 1 and 2. The profiles are extremely flat in the arc core, with sharp gradients existing near the wall. This behavior is due to the comparatively high thermal conductivities which characterize both gases at elevated temperatures and to the fact that radiation emission from the gases is intense and a strong function of temperature. Note that, for a prescribed set of arc operating conditions, the tempera-